Explicit Analytical Relations for Stress-controlled Rheodynamical Quantities in the case of Zener-Arrhenius Model

I. Underlying relations

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Aiming to develop a self-contained presentation regarding the thermorheodynamical behaviour of solid-like polymer materials, the Zener - Arrhenius model is used, the frequency or/and temperature dependences being considered in the case of stress-controlled conditions. Accordingly, the full set of the explicit general analytical relations for the primary characteristic thermorheodynamical quantities - the storage, loss and absolute compliances, as well as the loss factor - and the corresponding secondary ones - the storage, loss and absolute moduli - is obtained by using the appropriate rheological parameters including the low and high frequency limit values of the storage compliance, as well as the retardation time. These underlying analytical relations are provided in forms directly suitable for numerical simulation of frequency dependence in isothermal frequency - dependence, and as well for the temperature - dependence in isochronal circumstances.

Keywords: analytical relations, characteristic thermorheodynamical quantities, stress-controlled conditions, Zener - Arrhenius model, isothermal and isochronal circumstances

The use of well established conditions of rheological characterization provides distinct data concerning the peculiarities of mechanical response including the stiffness or/and deformation response of materials [1].

In the case of viscoelastic behaviour, the appropriate description framework of stiffness-like features needs the employ of strain-controlled conditions, resulting in a complete set of rheodynamical quantities including both the primary, modulus-like quantities, as well as the secondary ones, corresponding compliances [2-6].

Moreover, it is noteworthy to point out there is also the complementary framework of stress-controlled conditions, when the deformation-like response means the consideration of primary rheodynamical quantities containing the primary, compliance-like quantities, as well the secondary ones, corresponding moduli [7-17].

Consequently, in order to reveal the typical way in which the linear viscoelastic thermorheodynamical quantities arise and exhibit well defined dependences on frequency or/and temperature in isothermal and isochronal circumstances, respectively, the frame of a coupled Zener-Arrhenius model is considered.

The linear viscoelastic framework of solid-like behavior is evidenced by the use of the ε-σ rheological equation

$$q_0(T)\varepsilon + q_1(T)D_t\varepsilon = p_0(T)\sigma + p_1(T)D_t\sigma \qquad (1)$$

given in terms of ε and σ (the strain and stress, respectively) rheological variables and p_o, p_1 and q_o, q_1 nominal temperature dependent rheological parameters, the influence of time being included by the presence of time derivative operator, $D_t = \partial/\partial t$.

Consequently, the proposed model appear to be a three - parameter one, i. e.,

$$[1 + D_1(T)D_t]\varepsilon = [C_0(T) + C_1(T)D_t]\sigma \qquad (2.1)$$

the corresponding characteristic rheological parameters $C_o(T)$, $C_1(T)$, $D_1(T)$ being given as

$$C_0(T) = \frac{p_0(T)}{q_0(T)},\ C_1(T) = \frac{p_1(T)}{q_0(T)},\ D_1(T) = \frac{q_1(T)}{q_0(T)}\ . \qquad (3)$$

In the case of a controlled sinusoidal dynamic stress,

$$\sigma \equiv \tilde{\sigma} = \sigma_0 e^{i\omega t} \qquad (4)$$

where σ stands for the complex form of a sinusoidal excitation, σ_o is the strain amplitude and ω denotes the angular frequency of the given sinusoidal stress - the corresponding dynamic form of rheological equation results as ($D_t = i\omega$)

$$[1 + i\omega D_1(T)]\varepsilon = [C_0(T) + i\omega C_1(T)]\sigma \qquad (2.2)$$

The corresponding strain response - defined by the amplitude ε_o, the same frequency as that of the input stress, and delayed with the phase lag angle, δ_J, between the excitation and response - results from the complex deformation $\tilde{\varepsilon}$, as

$$\varepsilon = \tilde{\varepsilon} = \varepsilon_0(\omega,T)e^{i[\omega t - \delta_J(\omega,T)]} \qquad (5)$$

Consequently, by definition

$$\begin{aligned}\frac{\tilde{\varepsilon}}{\tilde{\sigma}} &= \frac{\varepsilon_0(\omega,T)}{\sigma_0}[\cos\delta_J(\omega,T) - i\sin\delta_J(\omega,T)] = \\ &= \tilde{J}(i\omega,T) \equiv J^*(\omega,T) = J'(\omega,T) - iJ''(\omega,T)\end{aligned} \qquad (6)$$

is the complex compliance, $\tilde{J}$, while *J'* and *J''* are the storage and loss compliance, respectively. On the other hand, for the absolute (viscoelastic) compliance, |J*|, results

$$|\,J^*(\omega,T)\,| = [J'^2(\omega,T) + iJ''^2(\omega,T)]^{1/2} \qquad (7)$$

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whereas $\beta_J=\tan\delta_J$ stands for the corresponding loss factor, being given as

$$\beta_J(\omega,T)=\tan\delta_J(\omega,T)=J''(\omega,T)/J'(\omega,T) \qquad (8)$$

Taking into account the fact that in the case of a σ-controlled excitation it is meaningful to interpret the quantities *J'*, *J''* as *direct primary* rheological quantities, $|J^*|$, β_J result to be the *derivate primary* ones. Furthermore, in order to have the complete set of σ-rheological quantities it is necessary to consider also the modulus-like quantities, i. e., the corresponding storage modulus, M'_J,

$$M'_J(\omega,T)=J'(\omega,T)/|J^*(\omega,T)|^2 \qquad (9)$$

loss modulus, M''_J,

$$M''_J(\omega,T)=J''(\omega,T)/|J^*(\omega,T)|^2 \qquad (10)$$

and absolute modulus, $|M^*_J|$,

$$|M^*{}_J(\omega,T)|=[M'^2_J(\omega,T)+M''^2_J(\omega,T)]^{1/2} \qquad (11)$$

which represent the *secondary* rheodynamical quantities.

Given the need to include in the above quoted rheodynamical quantities, in an explicit way the contribution of both frequency and temperature, it is suggested that - in principle, besides the frequency dependence, that of the temperature one - the stress rate could be given by employing the most frequently used exponential Arrhenius-like relation [18-22]. Thus, in a natural way, the σ - controlled retardation time, τ_σ is considered to be of the form

$$\tau_\sigma(T)=\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}} \qquad (12)$$

where $\tau_{\sigma\infty}=\tau_\sigma(\infty)$ stands for the prefactor (pre-exponential factor) - the value of the corresponding retardation time at infinite temperature (if $\theta_\sigma \neq 0$) - while $\theta_\sigma = A_\sigma/R$, i. e., the ratio of σ - Arrhenius activation energy, A_σ, to the universal gas constant, R=8.314 J/(mol*K) - indicates a "virtual"σ-temperature.

Accordingly, the complete "frequency and temperature" form of the rheodynamic equation is

$$[1+i\omega\tau_\sigma(T)]\varepsilon=[J'_l+i\omega J'_h\tau_\sigma(T)]\sigma \qquad (2.3)$$

where

$$J'_l\equiv J'(0)=C_0,\ J'_h\equiv J'(\infty)=\frac{C_1}{D_1},\ \tau_\sigma=D_1 \qquad (13)$$

the different rheological parameters being given in terms of the storage compliance at low (l) and high limit (h) frequency, respectively, as well as the σ - characteristic retardation time τ_σ.

Results and discussions

Due to the general form of the dynamic rheological equation (2. 3), it results that there are distinct dependences concerning the characteristic rheodynamical quantities on both frequency and temperature. In order to develop this approach from the standpoint of the complete set of evidenced rheodynamical quantities defined by the relations (6 - 11), it is necessary to draw the attention on essential circumstances including those of isothermal one - when the frequency dependence is considered - and of isochronal one - when the temperature dependence is examined.

Thus, the presentation of explicit analytical relations for a sinusoidal stress-controlled excitation needs, on the hand, the consideration of both the general definition of frequency and temperature dependences - (ω, T), and on the other hand, of those of characteristic forms - (ω; T) and (T; ω), respectively, when the frequency dependence is taken into account at given temperature (isothermal circumstances), and for the temperature dependence at fixed frequency (isochronal circumstances) [13].

The explicit analytical relations result as follows.

Direct primary quantities

The general expression of the *storage compliance*, obtained by using the relations (2), (6) and (13), is

$$J'(\omega,T)=\frac{C_0(T)+C_1(T)D_1(T)\omega^2}{1+D_1{}^2(T)\omega^2}=$$
$$=\frac{J'_l(T)+J'_h(T)\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{T}}\omega^2}{1+\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{T}}\omega^2} \qquad (14)$$

Consequently, in the case of frequency dependence at fixed temperature, $T=\underline{T}$, results

$$J'(\omega;\underline{T})=\frac{C_0(\underline{T})+C_1(\underline{T})D_1(\underline{T})\omega^2}{1+D_1{}^2(\underline{T})\omega^2}=\frac{C_{0\underline{T}}+C_{1\underline{T}}D_{1\underline{T}}\omega^2}{1+D_{1\underline{T}}{}^2\omega^2}=$$
$$=\frac{J'_{l\underline{T}}+J'_{h\underline{T}}\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2}{1+\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2} \qquad (14.\ 1)$$

and

$$J'(T;\underline{\omega})=\frac{C_0(T)+\underline{\omega}^2C_1(T)D_1(T)}{1+\underline{\omega}^2D_1{}^2(T)}=$$
$$=\frac{J'_l(T)+\underline{\omega}^2J'_h(T)\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{T}}}{1+\underline{\omega}^2\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{T}}} \qquad (14.\ 2)$$

for the temperature dependence at fixed frequency, $\omega=\underline{\omega}$.

- The general form of the *loss compliance* is, taking into account (2), (6) and (13),

$$J''(\omega,T)=\frac{[C_0(T)D_1(T)-C_1(T)]\omega}{1+D_1{}^2(T)\omega^2}=$$
$$=\frac{[J'_l(T)-J'_h(T)]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}}\omega}{1+\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{T}}\omega^2} \qquad (15)$$

For the ω - dependence at given temperature, $T=\underline{T}$,

$$J''(\omega;\underline{T})=\frac{[C_0(\underline{T})D_1(\underline{T})-C_1(\underline{T})]\omega}{1+D_1{}^2(\underline{T})\omega^2}=$$
$$=\frac{[J'_l(\underline{T})-J'_h(\underline{T})]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{\underline{T}}}\omega}{1+\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2} \qquad (15.1)$$

and

$$J''(T;\underline{\omega})=\underline{\omega}\frac{C_0(T)D_1(T)-C_1(T)}{1+\underline{\omega}^2D_1{}^2(T)}=$$
$$=\underline{\omega}\frac{[J'_l(T)-J'_h(T)]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}}}{1+\underline{\omega}^2\tau_{\sigma\infty}{}^2e^{\frac{2\theta_\sigma}{T}}} \qquad (15.2)$$

in the case of T - dependence at fixed frequency, $\omega=\underline{\omega}$.

Derivate primary quantities

- The general expression of the *absolute compliance,* as results from (2), (8) and (13), is

$$|J^{*}(\omega,T)|=[\frac{C_0^{\,2}(T)+C_1^{\,2}(T)\omega^2}{1+D_1^{\,2}(T)\omega^2}]^{1/2}=$$

$$=[\frac{J_l'^{2}(T)+J_h'^{2}(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2}{1+\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2}]^{1/2} \qquad (16)$$

Correspondingly, for the frequency dependence at given temperature, $T=\underline{T}$, one obtains

$$|J^{*}(\omega;\underline{T})|=[\frac{C_0^{\,2}(\underline{T})+C_1^{\,2}(\underline{T})\omega^2}{1+D_1^{\,2}(\underline{T})\omega^2}]^{1/2}=[\frac{C_{0\underline{T}}^{\,2}+C_{1\underline{T}}^{\,2}\omega^2}{1+D_{1\underline{T}}^{\,2}\omega^2}]^{1/2}=$$

$$=[\frac{J_{l\underline{T}}'^{\,2}+J_{h\underline{T}}'^{\,2}\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2}{1+\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2}]^{1/2} \qquad (16.1)$$

and

$$|J^{*}(T;\underline{\omega})|=[\frac{C_0^{\,2}(T)+\underline{\omega}^2C_1^{\,2}(T)}{1+D_1^{\,2}(T)\underline{\omega}^2}]^{1/2}=$$

$$=[\frac{J_{l\underline{T}}'^{\,2}+\underline{\omega}^2J_{h\underline{T}}'^{\,2}\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}}{1+\underline{\omega}^2\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2}]^{1/2} \qquad (16.2)$$

in the case of temperature dependence at given frequency, $\omega=\underline{\omega}$.

- The general expression of the *loss factor* is in virtue of (2), (8)and (13), by

$$\beta_J(\omega,T)\equiv\tan\delta_J(\omega,T)=\frac{[C_0(T)D_1(T)-C_1(T)]\omega}{C_0(T)+C_1(T)D_1(T)\omega^2}=$$

$$=\frac{[(J_l'(T)-J_h'(T)]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}}\omega}{J_l'(T)+J_h'(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2} \qquad (17)$$

while for the ω - dependence at fixed temperature, $T=\underline{T}$,

$$\beta_J(\omega;\underline{T})\equiv\tan\delta_J(\omega;\underline{T})=\frac{[C_0(\underline{T})D_1(\underline{T})-C_1(\underline{T})]\omega}{C_0(\underline{T})+C_1(\underline{T})D_1(\underline{T})\omega^2}=$$

$$=\frac{(C_{0\underline{T}}D_{1\underline{T}}-C_{1\underline{T}})\omega}{C_{0\underline{T}}+C_{1\underline{T}}D_{1\underline{T}}\omega^2}=\frac{(J_{l\underline{T}}'-J_{h\underline{T}}')\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{\underline{T}}}\omega}{J_{l\underline{T}}'+J_{h\underline{T}}'\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2} \qquad (17.1)$$

and

$$\beta_J(T;\underline{\omega})\equiv\tan\delta_J(T;\underline{\omega})=\underline{\omega}\frac{[C_0(T)D_1(T)-C_1(T)]}{C_0(T)+\underline{\omega}^2C_1(T)D_1(T)}=$$

$$=\underline{\omega}\frac{[J_l'(T)-J_h'(T)]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}}}{J_l'(T)+\underline{\omega}^2J_h'(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}} \qquad (17.2)$$

in the case of dependence at given frequency, $\omega=\underline{\omega}$.

Coresponding secondary quantities

- The general form of the *corresponding storage modulus* results, on the basis of (2), (9) and (13), as

$$M_J'(\omega,T)=\frac{C_0(T)+C_1(T)D_1(T)\omega^2}{C_0^{\,2}(T)+C_1^{\,2}(T)\omega^2}=$$

$$=\frac{J_l'(T)+J_h'(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2}{J_l'^{2}(T)+J_h'^{2}(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2} \qquad (18)$$

while for the frequency dependence at given temperature, $T=\underline{T}$, results

$$M_J'(\omega;\underline{T})=\frac{C_0(\underline{T})+C_1(\underline{T})D_1(\underline{T})\omega^2}{C_0^{\,2}(\underline{T})+C_1^{\,2}(\underline{T})\omega^2}=\frac{C_{0\underline{T}}+C_{1\underline{T}}D_{1\underline{T}}\omega^2}{C_{0\underline{T}}^{\,2}+C_{1\underline{T}}^{\,2}\omega^2}$$

$$=\frac{J_{l\underline{T}}'+J_{h\underline{T}}'\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2}{J_{l\underline{T}}'^{\,2}+J_{h\underline{T}}'^{\,2}\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2} \qquad (18.1)$$

and

$$M_J'(T;\underline{\omega})=\frac{C_0(T)+\underline{\omega}^2C_1(T)D_1(T)}{C_0^{\,2}(T)+\underline{\omega}^2C_1^{\,2}(T)}=$$

$$=\frac{J_l'(T)+\underline{\omega}^2J_h'(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}}{J_l'^{2}(T)+\underline{\omega}^2J_h'^{2}(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}} \qquad (18.2)$$

in the case of temperature dependence at given frequency, $\omega=\underline{\omega}$.

- The general expression of the *corresponding loss modulus* is, taking into account (2), (10) and(13), given by

$$M_J''(\omega,T)=\frac{[C_0(T)D_1(T)-C_1(T)]\omega}{C_0^{\,2}(T)+C_1^{\,2}(T)\omega^2}=$$

$$=\frac{[J_l'(T)-J_h'(T)]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}}\omega}{J_l'^{2}(T)+J_h'^{2}(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2} \qquad (19)$$

whereas for the ω - dependence at fixed temperature, $T=\underline{T}$,

$$M_J''(\omega;\underline{T})=\frac{[C_0(\underline{T})D_1(\underline{T})-C_1(\underline{T})]\omega}{C_0^{\,2}(\underline{T})+C_1^{\,2}(\underline{T})\omega^2}=\frac{[C_{0\underline{T}}D_{1\underline{T}}-C_{1\underline{T}}]\omega}{C_{0\underline{T}}^{\,2}+C_{1\underline{T}}^{\,2}\omega^2}$$

$$=\frac{(J_{l\underline{T}}'-J_{h\underline{T}}')\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{\underline{T}}}\omega}{J_{l\underline{T}}'^{\,2}+J_{h\underline{T}}'^{\,2}\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2} \qquad (19.1)$$

and

$$M_J''(T;\underline{\omega})=\underline{\omega}\frac{[C_0(T)D_1(T)-C_1(T)]}{C_0^{\,2}(T)+\underline{\omega}^2C_1^{\,2}(T)}=$$

$$=\underline{\omega}\frac{[J_l'(T)-J_h'(T)]\tau_{\sigma\infty}e^{\frac{\theta_\sigma}{T}}}{J_l'^{2}(T)+\underline{\omega}^2J_h'^{2}(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}} \qquad (19.2)$$

in the case of dependence at given frequency, $\omega=\underline{\omega}$.

- The general form of the *corresponding absolute modulus* results, given (2), (11) and (13), is

$$|M^{*}{}_J(\omega,T)|=[\frac{1+D_1^{\,2}(T)\omega^2}{C_0^{\,2}(T)+C_1^{\,2}(T)\omega^2}]^{1/2}=$$

$$=[\frac{1+\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2}{J_l'^{2}(T)+J_h'^{2}(T)\tau_{\sigma\infty}^{\,2}e^{\frac{2\theta_\sigma}{T}}\omega^2}]^{1/2} \qquad (20)$$

so that for the frequency dependence at given temperature, $T=\underline{T}$, one finds

$$|M^*{}_J(\omega;\underline{T})| = [\frac{1+D_1{}^2(\underline{T})\omega^2}{C_0{}^2(\underline{T})+C_1{}^2(\underline{T})\omega^2}]^{1/2} = [\frac{1+D_{1\underline{T}}{}^2\omega^2}{C_{0\underline{T}}{}^2+C_{1\underline{T}}{}^2\omega^2}]^{1/2}$$

$$= [\frac{1+\tau_{\sigma\infty}{}^2 e^{\frac{2\theta_\sigma}{\underline{T}}}\omega^2}{J'_{l\underline{T}}{}^2+J'_{h\underline{T}}{}^2\tau_{\sigma\infty}{}^2 e^{\frac{2\theta_\sigma}{T}}\omega^2}]^{1/2} \qquad (20.1)$$

and

$$|M^*{}_J(T;\underline{\omega})| = [\frac{1+\underline{\omega}^2 D_1{}^2(T)}{C_0{}^2(T)+\underline{\omega}^2 C_1{}^2(T)}]^{1/2} =$$

$$= [\frac{1+\underline{\omega}^2\tau_{\sigma\infty}{}^2 e^{\frac{2\theta_\sigma}{T}}}{J_l'^2(T)+\underline{\omega}^2 J_h'^2(T)\tau_{\sigma\infty}{}^2 e^{\frac{2\theta_\sigma}{T}}}]^{1/2} \qquad (20.2)$$

in the case of temperature dependence at given frequency, $\omega=\underline{\omega}$.

The explicit analytical relations obtained for the considered complete set of seven rheodynamical quantities provide a lot of consequences concerning the different forms of considered dependences as well as from the standpoint of characteristic, extremum (maximum and minimum) and inflection, points to be determined in the framework of numerical modeling.

In the case of *isothermal* circumstances, $T = \underline{T}$, the resulting frequency dependence, at stress-controlled conditions, reveals

- for the direct primary rheodynamical quantities - $J'(\omega;\underline{T})$ and $J''(\omega;\underline{T})$ ω the terms which are present in the characteristic ratios result to be of same order (2 to 2) for J' , whereas in the case of J" there is a dissimilar one, (1 to 2);

- for the derivate primary rheodynamical quantities - $|J^*(\omega;\underline{T})$ and $\beta_J(\omega;\underline{T})$ - the characteristic ratio is (2 to 2) in the case of $|J^*|$, and (1 to 2) for β_J;

- for the corresponding secondary rheodynamical quantities - $M'_J(\omega;\underline{T})$, $M''_J(\omega;\underline{T})$ and $|M^*_J(\omega;\underline{T})|$ - the values of characteristic ratios are (2 to 2) in the case of M'_J and $|M^*_J|$, while (1 to 2) it is obtained for M''_J.

On the other hand, in the case of *isochronal* circumstances, $\omega=\underline{\omega}$, at stress-controlled conditions, the temperature dependence points out:

- for the direct primary rheodynamical quantities, $J'(T;\underline{\omega})$ and $J''(T;\underline{\omega})$, the T exponentials appearing in the characteristic ratios reveals that in the case of J' the terms are of same order (2 to 2), and dissimilar ones (1 to 2) for J" ;

- for the derivate primary rheodynamical quantities - $|J^*(T;\underline{\omega})\ |$ and $\beta_J(T;\underline{\omega})$ - the order of exponential terms which appear in the characteristic ratios is (2 to 2) in the case of $|J^*|$, while the (1 to 2) result it is obtained for β_J;

- for the corresponding secondary rheodynamical quantities - $M'_J(T;\underline{\omega})$, $M''_J(T;\underline{\omega})$, and $M^*_J(T;\underline{\omega})|$- the characteristic ratios of exponential terms are given as (2 to 2) in the case of M'_J and $|M^*_J|$, and (1 to 2) for M''_J.

Conclusions

The set of explicit analytical relations obtained in the case of dynamic stress-controlled processes for both the primary and corresponding secondary rheological quantities reveals well definite qualitative features concerning the frequency or/and temperature variation trends.

Accordingly, it is pointed out the contribution of frequency dependence, in *isothermal* circumstances, while for the temperature dependence, in *isochronal* circumstances, the typical involvement of exponential terms containing the σ- activation energy (and the corresponding virtual temperature) are the active ones.

Moreover, regarding the direct thermorheodynamical quantities - the storage and absolute compliances - and the complementary ones - the storage and absolute compliances - respectively, there are typical frequency dependences, the situation being somewhat similar in case of temperature dependence.

Hereafter, even at a first glance, for the other evidenced direct thermorheodynamical quantities, including the loss compliance and the loss factor, as well as for the corresponding loss modulus, the frequency or temperature dependences come out as being analogous.

Distinct features of the full set of characteristic thermorheodynamical quantities in stress-controlled conditions remain to be illustrated in respect of noticeable frequency or/and temperature dependences by using the complex numerical simulation option.

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